Flux tube in stratified solar atmosphere: magnetohydrostatic equilibrium

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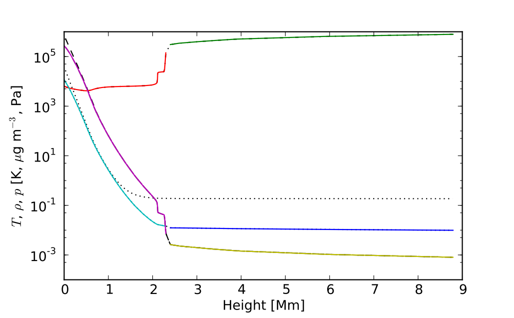
Atmospheric stratification

Title and background images courtesy SDO/NASA

Coronal heating problem?

(Vernazza et al. 1981)

(McWhirter et al. 1975)



Self-similar expanding flux tube and pressure balance

$$B_r = -\frac{\partial f}{\partial z} G, \quad B_\phi = 0, \quad B_z = \frac{\partial f}{\partial r} G, \tag{1}$$

By construction $\nabla \cdot \boldsymbol{B} = 0$ for arbitrary $f = f(r, \phi, z)$ or $G = G(f)$.
(Schlüter & Temesváry 1958, Deinzer 1965, Gibson & Low 1998, Schüssler & Rempel 2005, Gordovskyy & Jain 2007)

$$f = r b_{00} \exp\left(-\frac{z}{z_1}\right), \quad G = \frac{2}{\sqrt{\pi} f_0} \exp\left[-\left(\frac{f}{f_0}\right)^2\right] \tag{2}$$

$$\nabla p + \nabla \frac{|\boldsymbol{B}|^2}{2\mu_0} + (\boldsymbol{B} \cdot \nabla)\boldsymbol{B} = \rho \boldsymbol{g}, \tag{3}$$

(Fedun et al. 2011) Integrated numerically to find p and then ρ – turns out can be solved analytically. . .

Solving for corrections to p and ρ

$$\nabla(p_\mathrm{h} + p_\mathrm{m}) + \nabla\frac{|\boldsymbol{B}|^2}{2\mu_0} + (\boldsymbol{B}\cdot\nabla)\boldsymbol{B} = (\rho_\mathrm{h} + \rho_\mathrm{m})\boldsymbol{g}, \tag{4}$$

Separate hydrostatic and then correct for magnetohydrostatic effect.

$$p_\mathrm{h} = P_\mathrm{ref}(0) + g\int_0^z \rho_\mathrm{h}(z^*)\mathrm{d}z^* \tag{5}$$

Interpolation(Vernazza et al. 1981, McWhirter et al. 1975)

$$\frac{\partial p_\mathrm{m}}{\partial r} = -\frac{\partial}{\partial r}\frac{|\boldsymbol{B}|^2}{2\mu_0} - B_r\frac{\partial B_r}{\partial r} - B_z\frac{\partial B_r}{\partial z} \tag{6}$$

$$\frac{\partial p_\mathrm{m}}{\partial z} = \rho_\mathrm{m} g - \frac{\partial}{\partial z}\frac{|\boldsymbol{B}|^2}{2\mu_0} - B_r\frac{\partial B_z}{\partial r} - B_z\frac{\partial B_z}{\partial z} \tag{7}$$

(8)

Solving for corrections to p and ρ

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$$\frac{\partial p_\mathrm{m}}{\partial z} = \rho_\mathrm{m} g \quad -\frac{\partial}{\partial z}\frac{|\boldsymbol{B}|^2}{2\mu_0} - B_r\frac{\partial B_z}{\partial r} - B_z\frac{\partial B_z}{\partial z} \tag{7}$$

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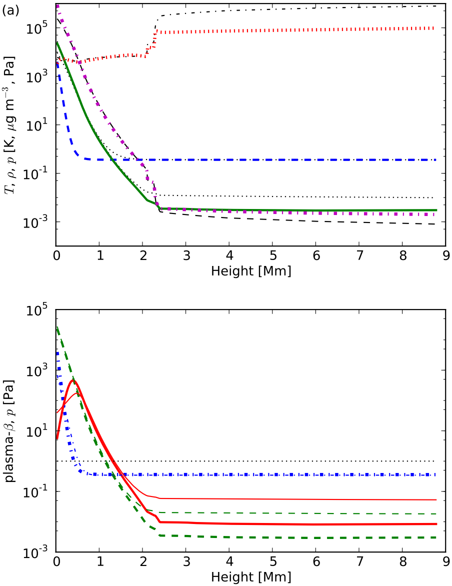
$$\rho_\mathrm{m} g - B_r\frac{\partial B_z}{\partial r} - B_z\frac{\partial B_z}{\partial z} + \frac{\partial}{\partial z}\left(\int B_r\frac{\partial B_r}{\partial r} + B_z\frac{\partial B_r}{\partial z}\mathrm{d}r\right) = 0 \tag{8}$$

Physical constraints



$$f = r\ b_{00} \exp\left(-\frac{z}{z_1}\right)$$

$$\text{plasma}-\beta = 2\mu_0 \frac{p}{|\boldsymbol{B}|^2}$$

(Aschwanden 2005, Khomenko et al. 2008)

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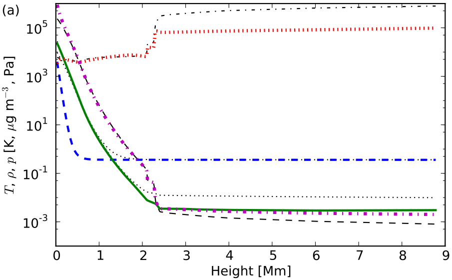
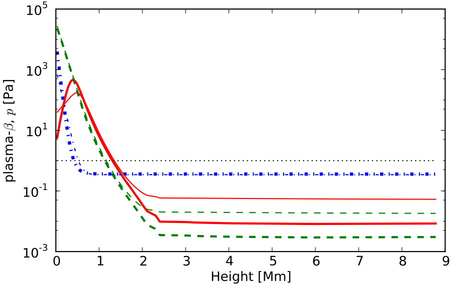


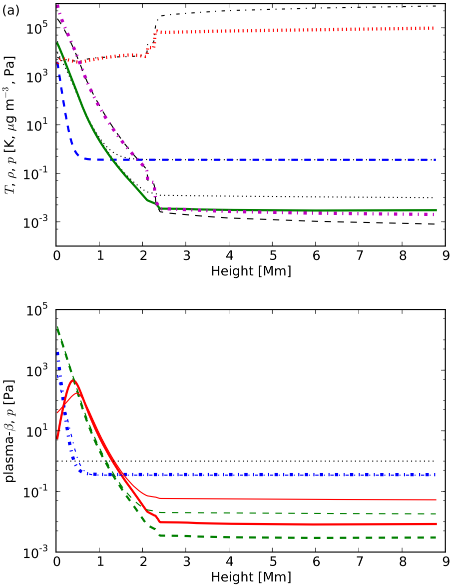


(Aschwanden 2005, Khomenko et al. 2008)

$$f = r\left[b_{01}\exp\left(-\frac{z}{z_1}\right) + b_{02}\exp\left(-\frac{z}{z_2}\right)\right]$$

$$\text{plasma}-\beta = 2\mu_0\frac{p}{|\boldsymbol{B}|^2}$$

Physical constraints



$$f = r\, B_{0z}$$

$$G = \frac{2}{\sqrt{\pi} f_0} \exp\left[-\left(\frac{f}{f_0}\right)^2\right]$$

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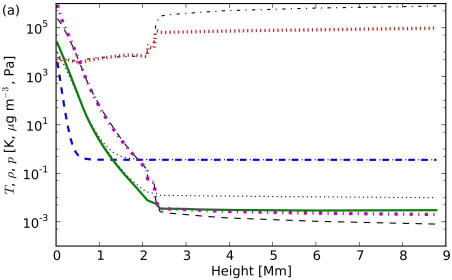
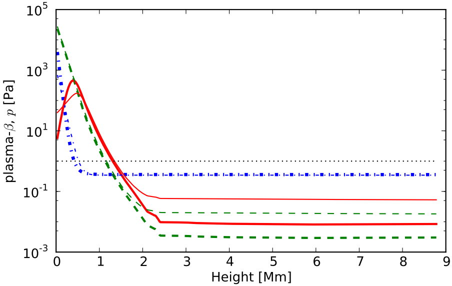


$$f = r\,B_{0z}$$

$$G' = \frac{2B_{0z}}{\sqrt{\pi}f_0}\exp\left[-\left(\frac{rB_{0z}}{f_0}\right)^2\right]$$



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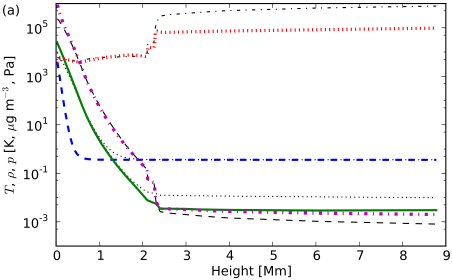

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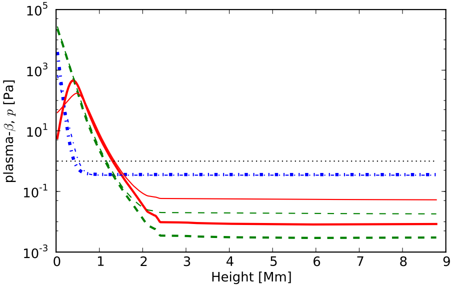

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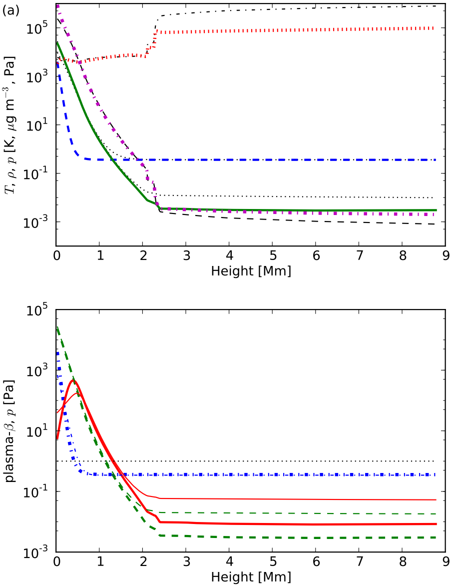

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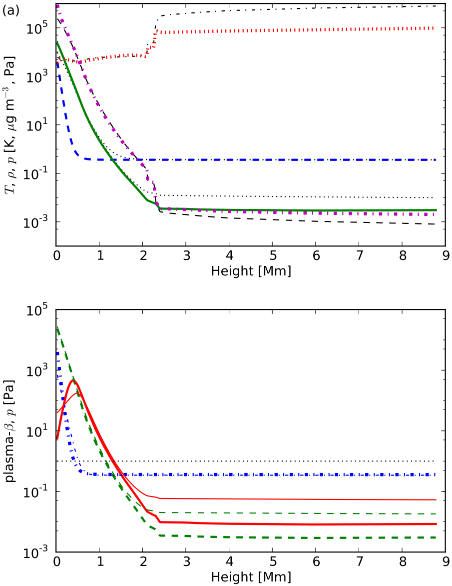

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$$B_r = -\frac{\partial f}{\partial z} B_{0z} G - \frac{\partial B_{0z}}{\partial z} \mathcal{G}, \quad G = \frac{\partial \mathcal{G}}{\partial f}$$

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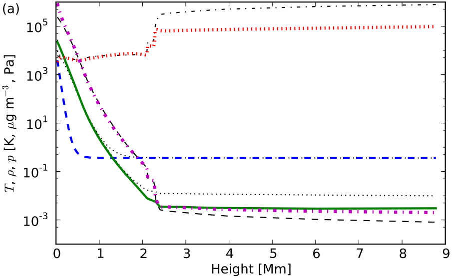
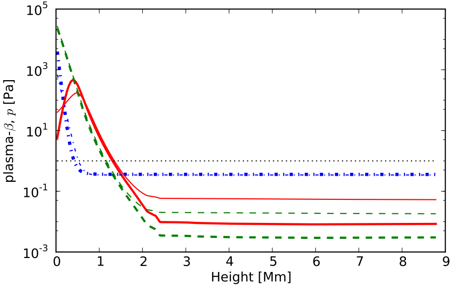
Physical constraints





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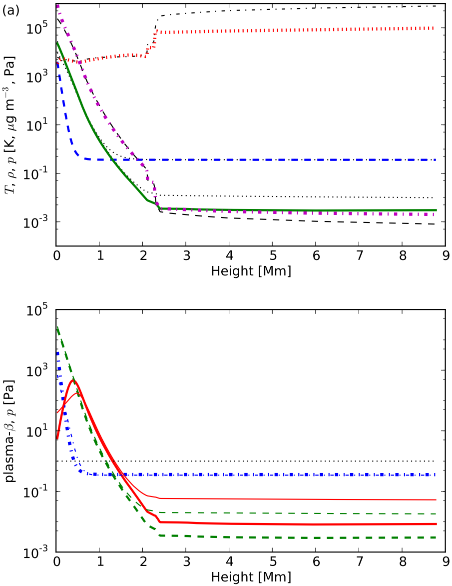

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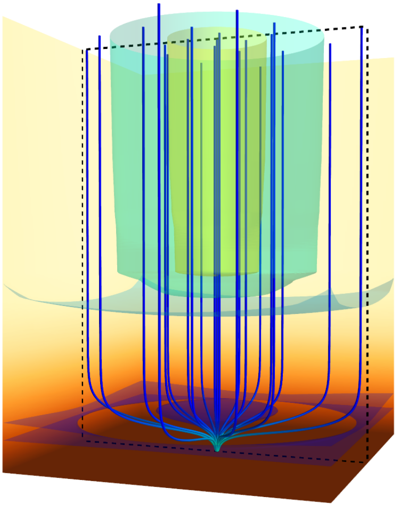
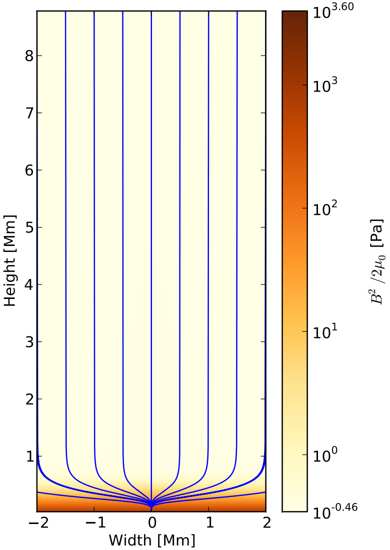
$$B_r = -\frac{\partial f}{\partial z} B_{0z} G - \frac{\partial B_{0z}}{\partial z} \mathcal{G} + \frac{r B_{bz}}{z_b}$$

$$B_z = \frac{\partial f}{\partial r} B_{0z} G + B_{bz}$$

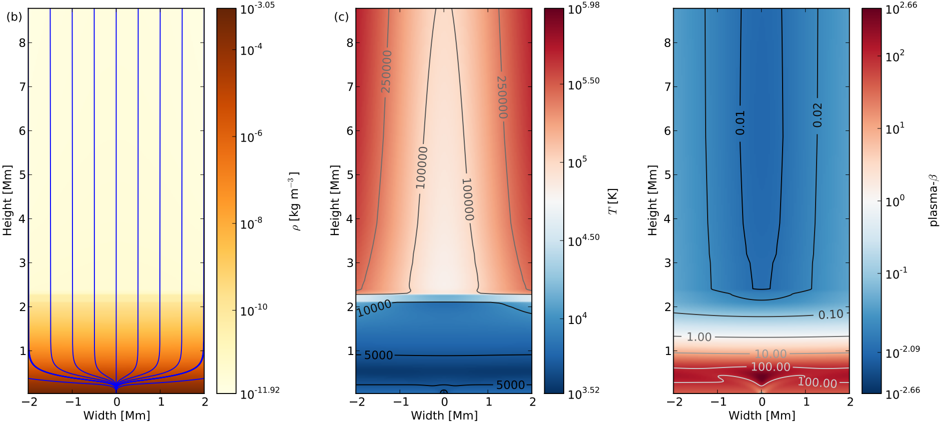
Single magnetic flux tube





B-field lines (blue), plasma$-\beta$ (purple – green isosurfaces), p_{thermal} (3D rear and bottom surfaces), p_{magnetic} (2D fill)

Single magnetic flux tube



ρ ($\boldsymbol{B}$-field lines – blue)

T

plasma$-\beta$

Single magnetic flux tube - pressure terms

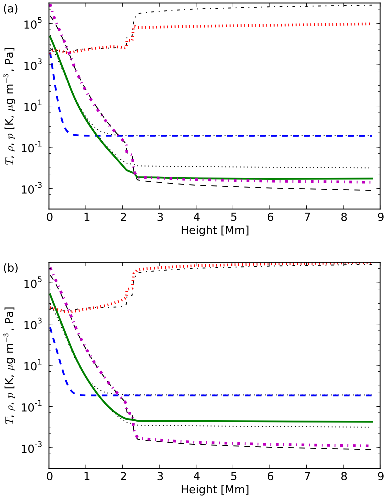
submitted MNRAS 17th May, 2013 `http://arxiv.org/abs/1305.4788`

$$
\begin{aligned}
p_{\mathrm{m}} &= P_{\mathrm{ref}}(z) - p_{\mathrm{h}}(z) - \frac{|\boldsymbol{B}|^2}{2\mu_0} \quad + \left(\frac{B_{bz} f^2 G}{B_{0z} z_b} - \frac{B_{bz} {f_0}^2 G}{2 B_{0z} z_b} \right) \frac{\partial B_{0z}}{\partial z} \\
&+ \left(\frac{B_{bz} f^2 G}{{B_{0z}}^2} - f \mathcal{G} G - \frac{{f_0}^2 G^2}{4} - \frac{\mathcal{G}^2}{2} \right) \frac{\partial B_{0z}}{\partial z}^2 \\
&+ \left(\frac{B_{0z} \mathcal{G}^2}{2} - \frac{B_{0z} {f_0}^2 G^2}{4} - \frac{B_{bz} {f_0}^2 G}{2 B_{0z}} \right) \frac{\partial^2 B_{0z}}{\partial z^2} \\
&- \left(\frac{{f_0}^2 B_{bz} G}{2 {z_b}^2} \right) + \left(B_{bz} \frac{\partial^2 B_{0z}}{\partial z^2} + \frac{B_{bz}}{z_b} \frac{\partial B_{0z}}{\partial z} \right) \int \mathcal{G} \mathrm{d}r \\
\int \mathcal{G} \mathrm{d}r &= \left\{ r \,\mathrm{erf}\left(\frac{f}{f_0} \right) + \frac{f_0}{\sqrt{\pi} B_{0z}} \left[\exp\left(-\frac{f^2}{{f_0}^2} \right) - 1 \right] \right\} \\
&= r \mathcal{G} + \frac{{f_0}^2 G}{2 B_{0z}} - \frac{f_0}{\sqrt{\pi} B_{0z}}.
\end{aligned}
$$

Single magnetic flux tube - density terms

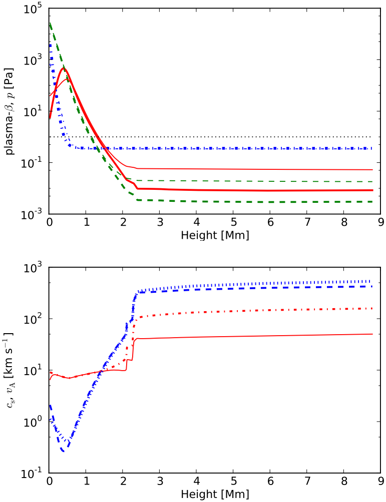
$$
\begin{aligned}
\int \mathrm{d}z \quad \rho_{\mathrm{m}} g \quad & - \quad 2{B_{0z}}^3 G \left[\frac{f\mathcal{G}}{{f_0}^2} + G \right] \frac{\partial B_{0z}}{\partial z} + \frac{B_{bz}}{{z_b}^2} \mathcal{G} r \frac{\partial B_{0z}}{\partial z} + \frac{2{B_{0z}}^4 B_{bz} G r^2}{{f_0}^2 z_b} \\
& - \quad 2 B_{bz} B_{0z} G \left[1 - \frac{f^2}{{f_0}^2} \right] \frac{\partial B_{0z}}{\partial z} - \frac{B_{bz}}{{z_b}^2} \frac{f_0 z_{\mathrm{h}}}{\sqrt{\pi} B_{0z}} \frac{\partial B_{0z}}{\partial z} \\
& + \quad \left[f\mathcal{G}G + \frac{\mathcal{G}^2}{2} - 3 B_{bz} G r^2 - \frac{B_{bz} f_0 z_{\mathrm{h}}}{\sqrt{\pi} {B_{0z}}^2} \right] \frac{\partial B_{0z}}{\partial z} \frac{\partial^2 B_{0z}}{\partial z^2} + \frac{{B_{bz}}^2}{z_b} \\
& + \quad \left[\frac{3{f_0}^2 G^2}{4} - f^2 G^2 \right] \frac{\partial B_{0z}}{\partial z} \frac{\partial^2 B_{0z}}{\partial z^2} - \frac{B_{bz} f G r}{z_b} \frac{\partial^2 B_{0z}}{\partial z^2} \\
& + \quad \frac{B_{bz}}{z_b} \left[\frac{2 f^2 G r^2}{{f_0}^2} - G r^2 - \frac{f_0 z_{\mathrm{h}}}{\sqrt{\pi} {B_{0z}}^2} \right] {\frac{\partial B_{0z}}{\partial z}}^2 + \frac{{B_{0z}}^2 B_{bz} G}{z_b} \\
& + \quad 2 G r \left[\mathcal{G} - \frac{f^2 \mathcal{G}}{{f_0}^2} + \frac{B_{bz} f r^2}{{f_0}^2} \right] {\frac{\partial B_{0z}}{\partial z}}^3 + \frac{B_{0z} {f_0}^2 G^2}{4} \frac{\partial^3 B_{0z}}{\partial z^3} \\
& + \quad \left[\frac{B_{bz} f_0 z_{\mathrm{h}}}{\sqrt{\pi} B_{0z}} - \frac{B_{0z} \mathcal{G}^2}{2} - B_{bz} \mathcal{G} r \right] \frac{\partial^3 B_{0z}}{\partial z^3} - \frac{B_{bz} {f_0}^2 G}{2 {z_b}^3} = 0.
\end{aligned}
$$

Flux tube structure



Upper: along flux tube axis, lower: outside flux tube

[T-red, ρ-purple, p_t-green, p_m-blue]

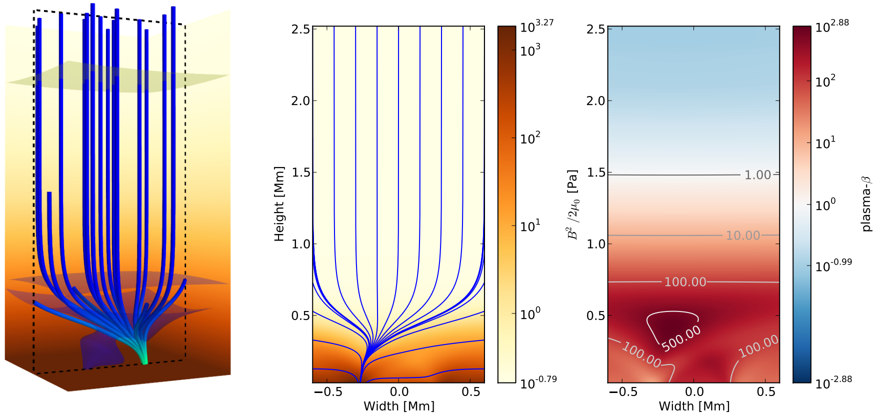


Upper: plasma-β-red, p_t-green, p_m-blue,

Lower: c_s-red, v_A-blue

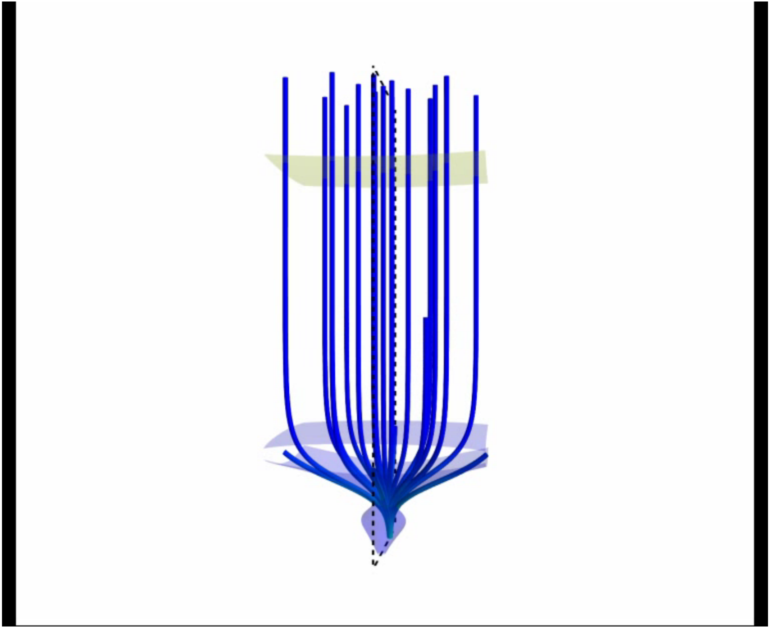
[axis (think) outside (thin)]

Multiple magnetic flux tube



B-field lines (blue), plasma $-\beta$ (purple – green isosurfaces), p_{thermal} (3D rear and bottom surfaces), p_{magnetic} (2D fill), plasma-β (red – blue)

Multiple magnetic flux tube



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