Multiphase interstellar medium: identifying the mean field

Fred Gent [1]
Maarit Mantere [2]
Anvar Shukurov [1]
Graeme Sarson [1]
Andrew Fletcher [1]

Department of Mathematics, Newcastle University, Newcastle upon Tyne NE1 7RU, UK

Department of Physics, University of Helsinki, FI-00014, Finland

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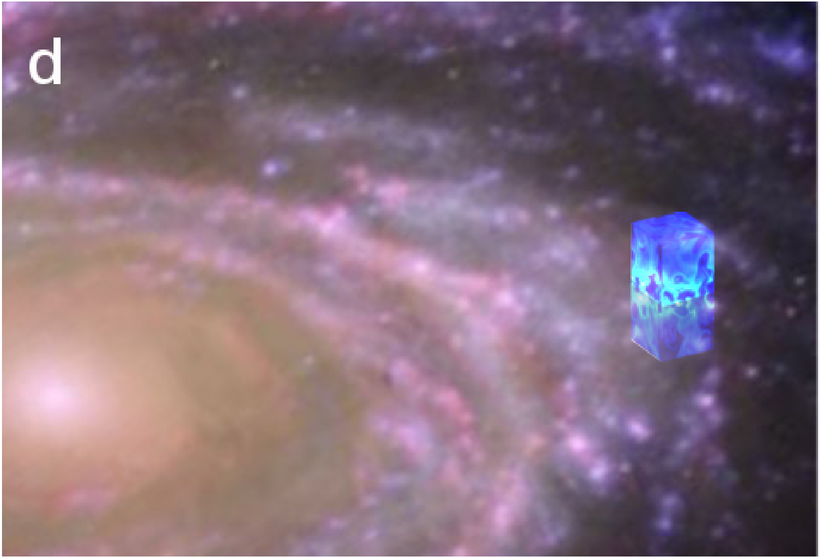
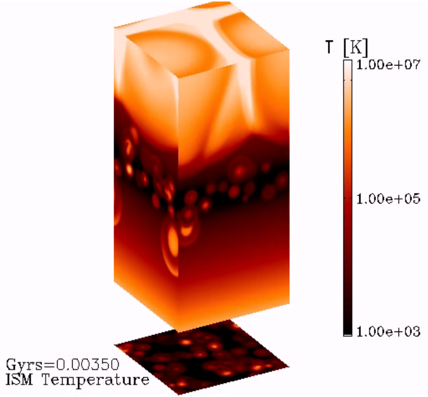
Modelling the interstellar medium in rotating galaxies

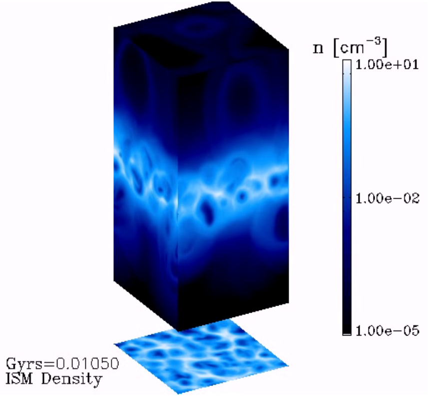


ISM SN dynamics: 3D simulation

Temperature



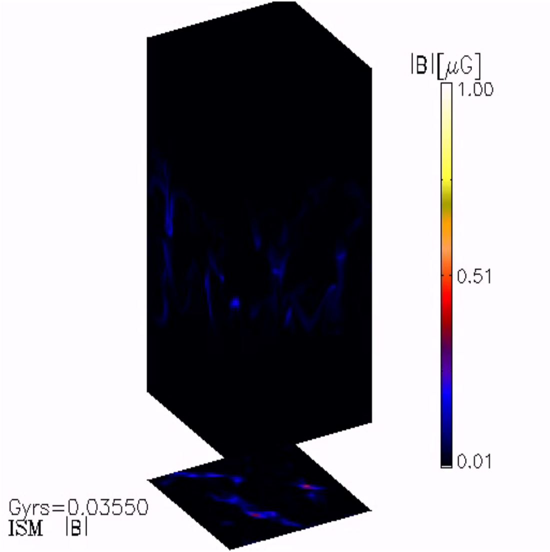
Gas Density



Composition of the ISM a multi-phase environment
(Cox & Smith 1974, McKee & Ostriker 1977)

Differential cooling – compressions

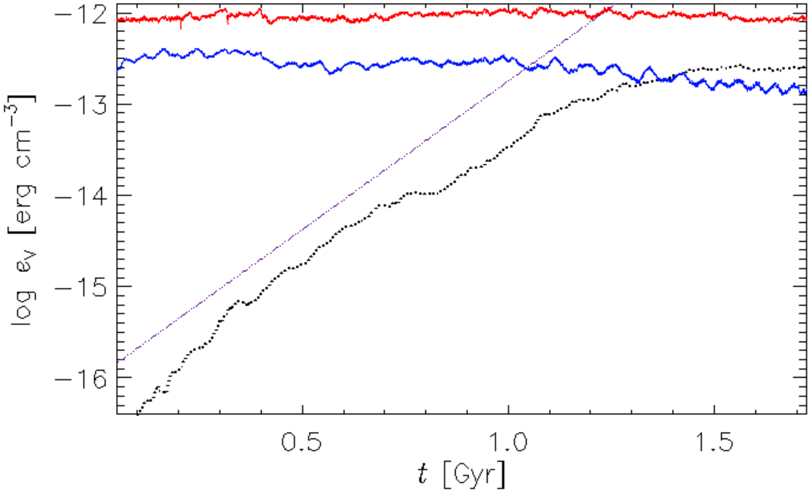
Hunting a galactic dynamo



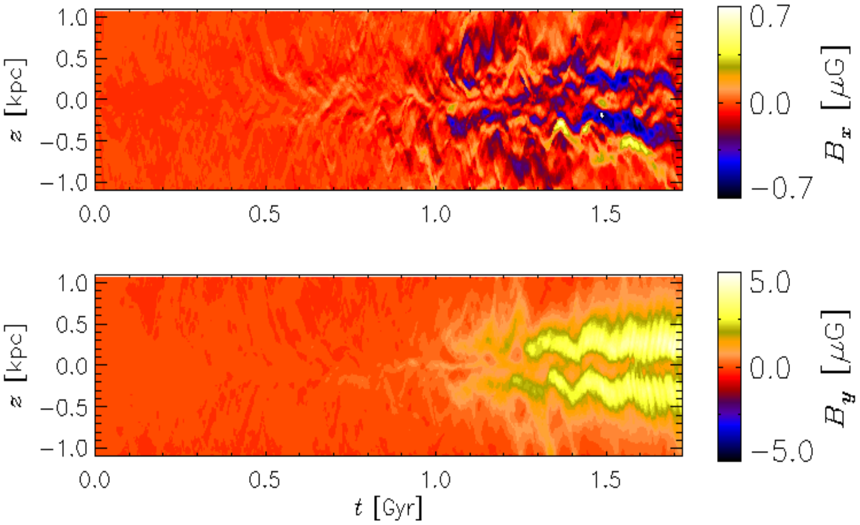
non-ideal MHD

- Differential rotation
- SN - sources of vorticity and random velocity
  - shocks
  - density gradient
  - turbulence
- Azimuthal seed ***B***
- Magnetic diffusivity

Comparing thermal, kinetic and magnetic energy densities



Horizontal averages of B_x and B_y over time



Mean field, fluctuation or both?

How do we identify the mean field?

$$\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{b},$$

$\boldsymbol{B}_0$ regular (mean) field,

$\boldsymbol{b}$ random (fluctuating) field,

such that $\langle \boldsymbol{b} \rangle = \mathbf{0}$.

- Volume averaging
- Horizontal averaging
  (Gressel 2008)
  (Dobbs & Price 2008)

  non-galaxy applications

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  non-galaxy applications

- Applying a low pass filter $G(\boldsymbol{x}, l)$
  (Germano 1992) ref turbulent $\boldsymbol{u}$
- Convolve $\boldsymbol{B}$ with gaussian

$$\boldsymbol{B}_0(\boldsymbol{x}, l_B) = \int_V \boldsymbol{B}(\boldsymbol{x}')G(\boldsymbol{x}, \boldsymbol{x}', l_B)d\boldsymbol{x}'$$

$$G(\boldsymbol{x}, \boldsymbol{x}', l_B) = \frac{1}{(\sqrt{2\pi} l_B)^3} \exp\left\{\frac{-(\boldsymbol{x} - \boldsymbol{x}')^2}{2\, l_B^2}\right\}$$

$$\hat{\boldsymbol{B}}_0(\boldsymbol{k}, l_B) = \hat{\boldsymbol{B}}(\boldsymbol{k}) \exp\left\{-2(l_B\pi)^2 \boldsymbol{k} \cdot \boldsymbol{k}\right\}$$

$$\hat{\boldsymbol{B}}_0(\boldsymbol{k}, l_B) = \mathcal{F}\left[\boldsymbol{B}_0(\boldsymbol{x}, l_B)\right]$$

What is the appropriate filter?

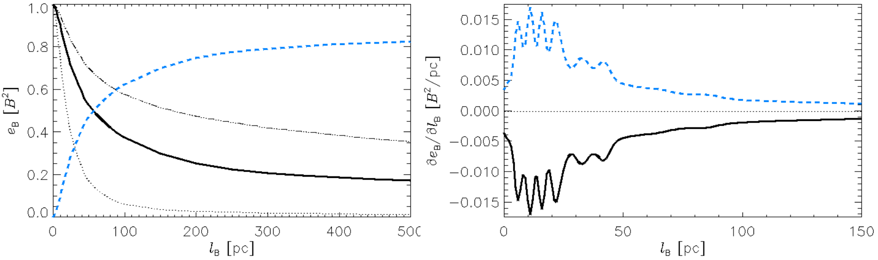
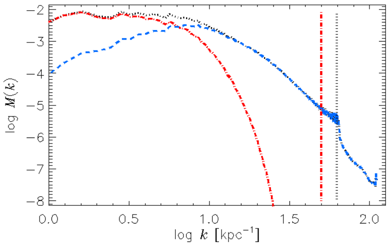


Figure: $e_{\mathrm{B}0}$ (black); e_{b} (blue), normalized by $[B^2]$ vs separation length l_B

Correlation length of ISM turbulence near mid-plane $l_0 \approx 60 - 100\,\mathrm{pc}$ (Korpi et al. 1999, Gent et al. 2012).

Scale separation!

$l_B = 20\,\mathrm{pc}$



- e_B black
  e_B0 red
  e_b blue
- vertical red line $k_B = l_B^{-1}$
- vertical black line $k_{\Delta x} = \Delta x^{-1}$

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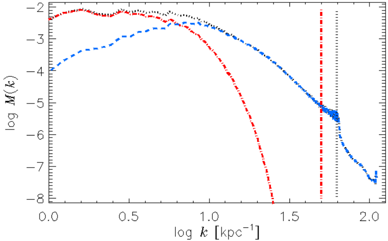

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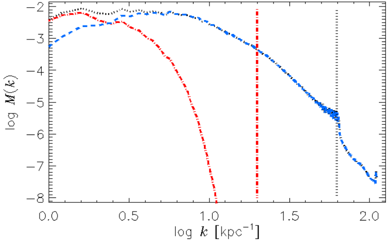
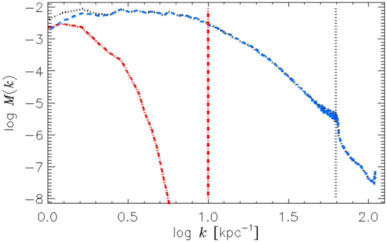
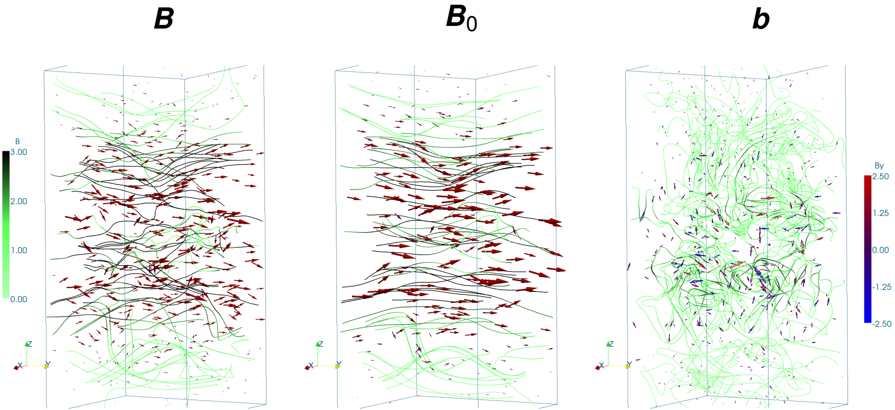
$l_B = 50\,\mathrm{pc}$



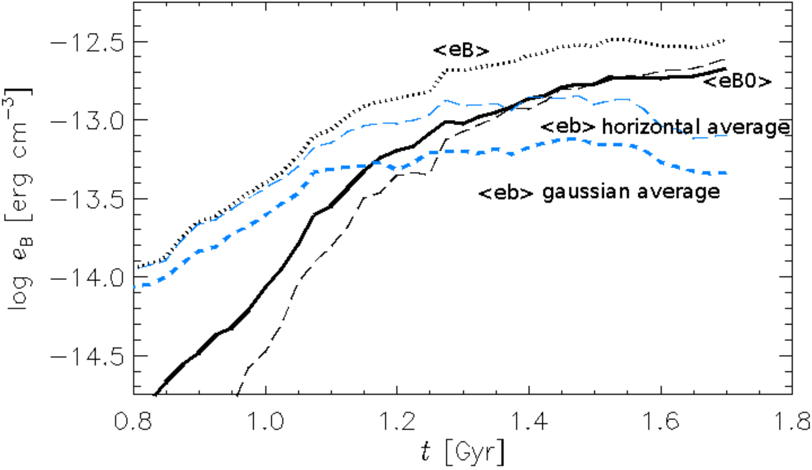
$l_B = 100\,\mathrm{pc}$



3D Structure of the total, mean and random ***B***-Field



Horizontal averaging vs gaussian smoothing



Averaging with gaussian smoothing does not obey Reynolds rules

- $\boldsymbol{B}_0 = \boldsymbol{B} * G(\boldsymbol{x}, l)$, but $\boldsymbol{B}_0 \neq \boldsymbol{B}_0 * G(\boldsymbol{x}, l)$

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- Non-vanishing cross terms

$$B^2 = B_0^2 + 2\boldsymbol{B}_0 \cdot \boldsymbol{b} + b^2$$

- $\boldsymbol{B}_0 \cdot \boldsymbol{b} \equiv 0$ satisfied for constant $\boldsymbol{B}_0$ over whole volume

(horizontal average special case of this)

$$\begin{aligned} \boldsymbol{B}_0 \cdot \boldsymbol{b} =& \boldsymbol{B} G(\boldsymbol{x}, l) \cdot \boldsymbol{B}[1 - G(\boldsymbol{x}, l)] \\ =& B^2 (1 - G) G \geq 0 \qquad \forall \parallel G \parallel_2 \neq 1 \end{aligned}$$

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- a rational alternative formulation: $b^2 = |B|^2 - |B_0|^2$

Eyink G 2012 'Course Notes for 550.693: Turbulence Theory Spring 2012 ' www.ams.jhu.edu/~eyink/Turbulence/notes.html.

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Summary and discussion

- Why is it important to define the mean and random properties of the velocity and magnetic fields?
- What is the correct physical approach to defining the mean field? – and does it affect the result?
- How do we identify the correct l_B to define the mean field?

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Equations

$$\frac{D\rho}{Dt} = -\nabla \cdot (\rho \boldsymbol{u}) + \dot{\rho}_{\rm SN}, \quad (1)$$

$$\begin{aligned}\frac{D\boldsymbol{u}}{Dt} = & -\rho^{-1}\nabla\sigma_{\rm SN} - c_{\rm s}^2\nabla\left(s/c_p + \ln\rho\right) - \nabla\Phi - Su_x\hat{\boldsymbol{y}} \\ & - 2\boldsymbol{\Omega}\times\boldsymbol{u} + \rho^{-1}\boldsymbol{j}\times\boldsymbol{B} + \zeta_\nu\left(\nabla\nabla\cdot\boldsymbol{u}\right) \\ & + \nu\left(\nabla^2\boldsymbol{u} + \tfrac{1}{3}\nabla\nabla\cdot\boldsymbol{u} + 2\mathsf{W}\cdot\nabla\ln\rho\right), \end{aligned} \quad (2)$$

$$\begin{aligned}\rho T\frac{Ds}{Dt} = & \dot{\sigma}_{\rm SN} + \rho\Gamma - \rho^2\Lambda + \nabla\cdot\left(c_p\rho\chi\nabla T\right) + \eta\mu_0\boldsymbol{j}^2 \\ & + 2\rho\nu\left|\mathsf{W}\right|^2 + \zeta_\chi\left(\rho T\nabla^2 s + \nabla\ln\rho T\cdot\nabla s\right) + \rho T\nabla\zeta_\chi\cdot\nabla s, \end{aligned} \quad (3)$$

$$\frac{D\boldsymbol{A}}{Dt} = \boldsymbol{u}\times\boldsymbol{B} + (\eta + \zeta_\eta)\nabla^2\boldsymbol{A} + (\nabla\cdot\boldsymbol{A})\nabla\zeta_\eta, \quad (4)$$

Notation

	mass equation	ζ_ν	shock-capturing viscosity
ρ	gas density	ν	viscosity
t	time	$\mathbf{W}$	rate of strain tensor
$\boldsymbol{u}$	gas velocity		*energy equation*
$\dot{\rho}_{\rm SN}$	rate of SN mass ejecta	T	temperature
	Navier Stokes	$\rho\Gamma$	uv-heating
$\dot{\sigma}_{\rm SN}$	rate of SN energy injection	$\rho^2\Lambda$	radiative cooling
$c_{\rm s}$	adiabatic speed of sound	χ	thermal conductivity
s	specific entropy	η	magnetic diffusivity
c_p	heat capacity at constant p	μ_0	magnetic vacuum permeability
S	velocity shear rate	$\|\mathbf{W}\|$	determinant of $\mathbf{W}$
$\boldsymbol{\Omega}$	Galactic rotation angular velocity	ζ_χ	shock-capturing χ
Φ	gravitational potential		*induction equation*
$\boldsymbol{B}$	magnetic flux density	$\boldsymbol{A}$	magnetic vector potential
$\boldsymbol{j}$	current density	ζ_η	shock-capturing η