Resolving the Field Length in the interstellar medium

Fred Gent [1]
Maarit Korpi [2]
Anvar Shukurov [1] , Graeme Sarson [1] , Andrew Fletcher [1]

Department of Mathematics, Newcastle University, Newcastle upon Tyne NE1 7RU, UK

Department of Physics, University of Helsinki, FI-00014, Finland

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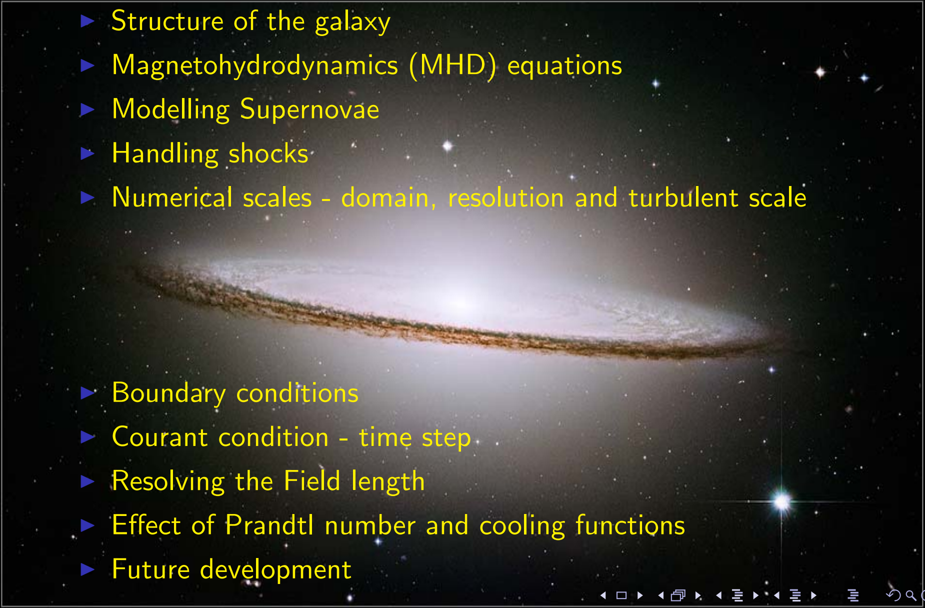

- Structure of the galaxy
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- Modelling Supernovae
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Structure of the galaxy

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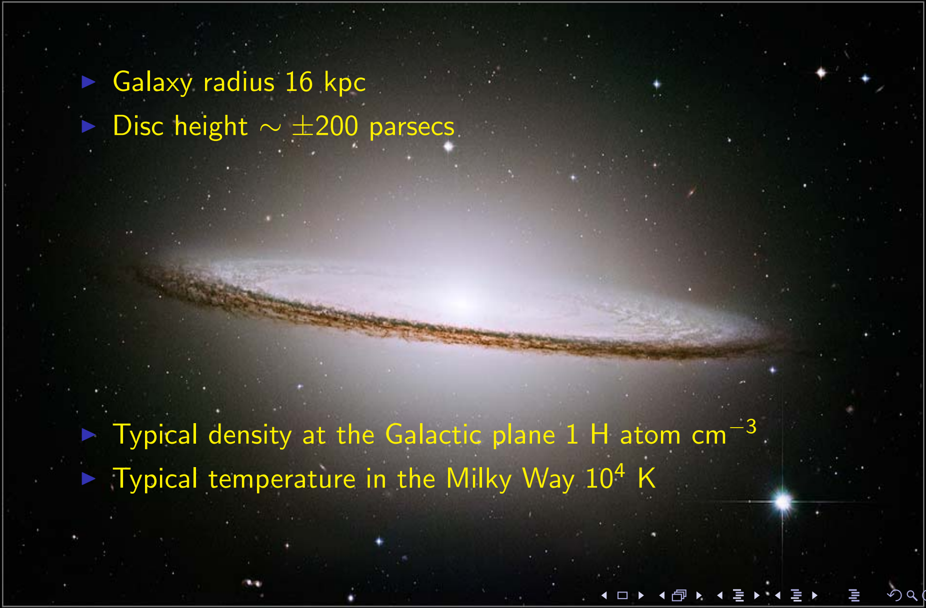


- Galaxy radius 16 kpc
- Disc height $\sim \pm 200$ parsecs
- Typical density at the Galactic plane 1 H atom cm^{-3}
- Typical temperature in the Milky Way 10^4 K

Whirlpool Galaxy · M51

- Angular momentum of the Galaxy 25 km s^{-1}
- Differential rotation
- Circulation in the Solar neighbourhood 200 Myrs

The equations

- The continuity equation

$$\frac{D\ln\rho}{Dt} = -\nabla \cdot \mathbf{u}$$

- The equation of state

$$p = \frac{k_\mathrm{B}}{\mu m_\mathrm{u}} \rho T$$

- The induction equation

$$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{u} \times \mathbf{B} - \eta \mu_0 \mathbf{j}$$

k_B
Boltzmann constant

m_u
atomic mass

μ
mean molecular weight

η
magnetic diffusivity

μ_0
magnetic permeability

The momentum equation

- The momentum equation

$$\frac{D\mathbf{u}}{Dt} = -c_s^2\nabla\left(\frac{s}{c_p} + \ln\rho\right) - \nabla\Phi_{\rm grav} - Su_x\hat{\mathbf{y}}$$
$$- \quad 2\Omega\times\mathbf{u} \quad + \quad \frac{\mathbf{j}\times\mathbf{B}}{\rho} \quad + \quad \zeta\left(\nabla\nabla\cdot\mathbf{u}\right)$$
$$+ \quad \nu\left(\nabla^2\mathbf{u} + \frac{1}{3}\nabla\nabla\cdot\mathbf{u} + 2\mathbf{S}\cdot\nabla\ln\rho\right),$$

pressure
gravity
Shear
Rotation
Lorentz
shock viscosity

where the advective derivative,

$$\frac{D\mathbf{u}}{Dt} = \frac{\partial}{\partial t} + (\mathbf{u}_0 + \mathbf{u})\cdot\nabla,$$

includes the transport by the imposed shear flow $\mathbf{u}_0 = (0, Sx, 0)$.

The entropy equation

- The entropy equation

$$\rho T \frac{Ds}{Dt} = \sigma_{\rm SN} + \rho\Gamma - \rho^2\Lambda + \nabla\cdot(K\nabla T) + \eta\mu_0\mathbf{j}^2 + 2\rho\nu\mathbf{S}^2 + \zeta\rho(\nabla\cdot\mathbf{u})$$

Super Novae

UV heating

radiative cooling

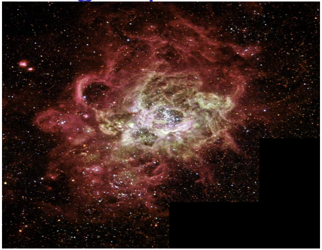
Ohmic heating

shock viscosity

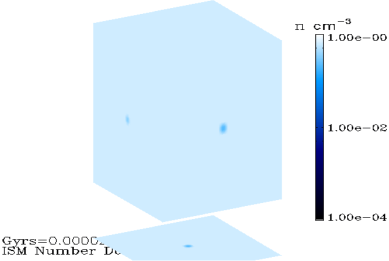
Interstellar flows are highly compressible, so we need to apply a shock viscosity ζ and include the traceless rate-of-strain tensor $\mathbf{S}$, such that

$$2S_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\nabla\cdot\mathbf{u}.$$

Modelling Supernovae - Handling Shocks

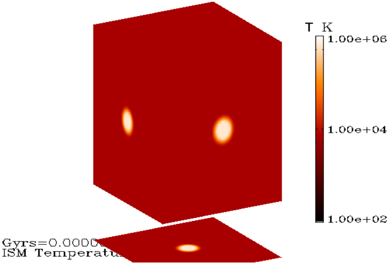


Observed SN remnant

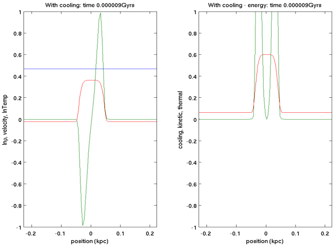


Mass image of remnant

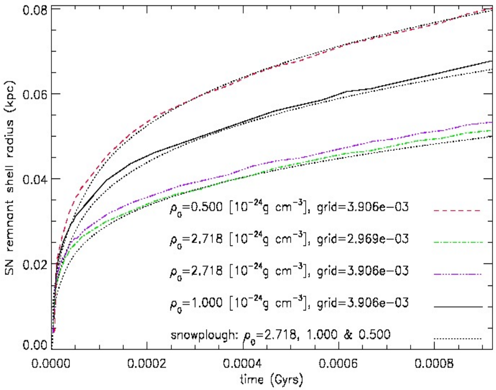
Thermal image of remnant



Shock profile of remnant



Modelling Supernovae - Results



$$R = \left(\frac{25}{3\pi}\right)^{\frac{1}{5}} \left(\frac{E}{\rho_0}\right)^{\frac{1}{5}} t^{\frac{2}{5}}$$

(adiabatic)

$$R = R_{tr}\left(1 + 4\frac{\dot{R}_{tr}}{R_{tr}}(t - t_{tr})\right)^{\frac{1}{4}}$$

(snowplough)

$$\dot{R}_{tr} \propto \rho_0^{2/17} E^{1/17}$$

((Woltjer 1972):transition $\dot{R}_{tr}$)

Numerical Scales

Numerical domain: $0.5 \times 0.5 \times 2$ kpc^3 Resolution: 4.17^3 pc^3

Array: $120 \times 120 \times 480$ 96 procs 200000 cpu hrs

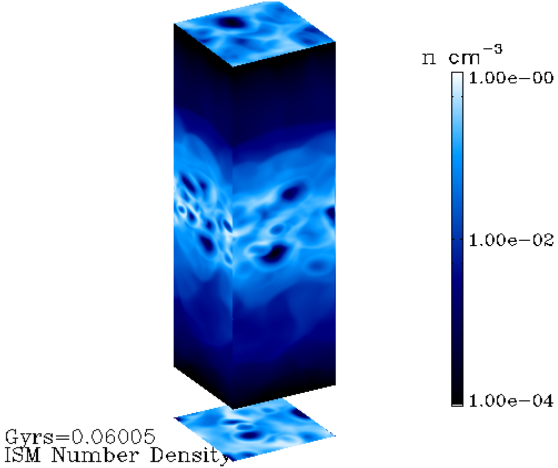


Figure: Density dynamics of simulation

- Dynamical Scales
- SN Remnants radii $60 \sim 200$ pc
- Disc scale height ~ 200 pc
- Cold dense clouds ~ 10 pc
- sound speed $c_s = 10 \sim 300$ km s^{-1}

Boundary Conditions

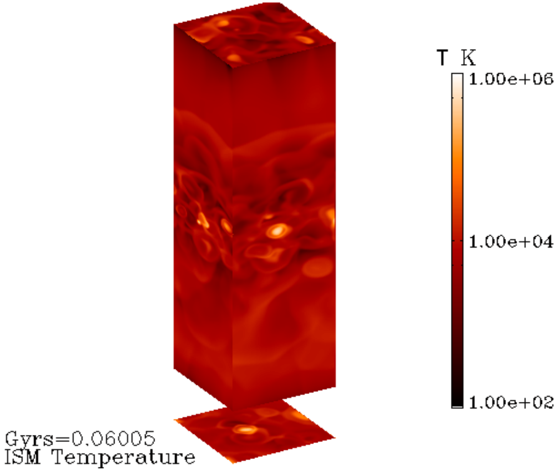


Figure: thermal dynamics of simulation

- Ghost zones
- Sliding periodic boundary
- Approx radial/azimuthal
- Dynamical scale $\ll$ width
- Outflow vertical boundary
- Infinite inflows? mass conservation
- Thermal spikes - dynamically dominant disc
- Galactic fountain

Time step stability

$$\text{Courant condition}: \frac{|\mathbf{u}|\ \delta t}{\delta x} \leq c$$

$$\delta t = \min\left(c_{\delta t}\frac{\delta x_{\rm min}}{U_{\rm max}}, c_{\delta t,\rm v}\frac{\delta x_{\rm min}^2}{D_{\rm max}}, c_{\delta t,\rm s}\frac{1}{H_{\rm max}}\right);\ \delta x_{\rm min} \equiv \min(\delta x, \delta y, \delta z);$$

$$U_{\rm max} \equiv \max\left(|\mathbf{u}| + \sqrt{c_{\rm s}^2 + v_{\rm A}^2}\right),$$

$c_{\rm s}$: sound speed; $v_{\rm A}$: Alfvén speed

$$D_{\rm max} = \max(\nu, \gamma\chi, \eta, D),$$

ν: kinematic viscosity; $\chi = K/(c_p\rho)$: thermal diffusivity; η: magnetic diffusivity

$$H_{\rm max} = \max\left(\frac{2\nu\mathbf{S}^2 + \zeta_{\rm shock}(\nabla\mathbf{u})^2 + \mathcal{H} + \sigma_{\rm SN}}{c_v T}\right)$$

Mesh Reynolds Number & Field Length

Mesh Reynolds Number

$$\mathrm{Re}_{\mathrm{mesh}} = \frac{\max(|\mathbf{u}|)\max(\delta x, \delta y, \delta z)}{\nu}$$

Field Length

$$\chi_{\mathrm{min}} = \frac{(1-\beta)\,\tau_{\mathrm{cool}}}{\gamma}\left(\frac{\delta x}{2\pi}\right)^2,$$

τ_{cool}: inverse of cooling time scale; β exponent in the thermally unstable temperature phase

$$\mathbf{\Lambda} = \Lambda_i T^{\beta_i},\ \mathrm{where\ T_i \leq T \leq T_{i+1}}$$

Stable computation dependent on Temperature scales $10 - 10^7$ K

$$\nu \propto \sqrt{T}; \chi \propto \sqrt{T}$$

Cooling functions

T_k [K]	$\Lambda_k [\mathrm{erg\ g^{-2} s^{-1} cm^3 K^{-fi_k}}]$	β_k
10	3.70×10^{16}	2.12
141	9.46×10^{18}	1.00
313	1.18×10^{20}	0.56
6102	1.10×10^{10}	3.21
10^5	1.24×10^{27}	-0.20
2.88×10^5	2.39×10^{42}	-3.00
4.73×10^5	4.00×10^{26}	-0.22
2.11×10^6	1.53×10^{44}	-3.00
3.98×10^6	1.61×10^{22}	0.33
2.0×10^7	9.23×10^{20}	0.50

T_k [K]	$\Lambda_k [\mathrm{erg\ g^{-2} s^{-1} cm^3 K^{-fi_k}}]$	β_k
10	9.88×10^5	6.000
300	8.36×10^{15}	2.000
2000	3.80×10^{17}	1.500
8000	1.76×10^{12}	2.867
10^5	6.76×10^{29}	-0.650
10^6	8.51×10^{22}	0.500

Table: Cooling function: RB (lower) SS (top).

Cooling function curves

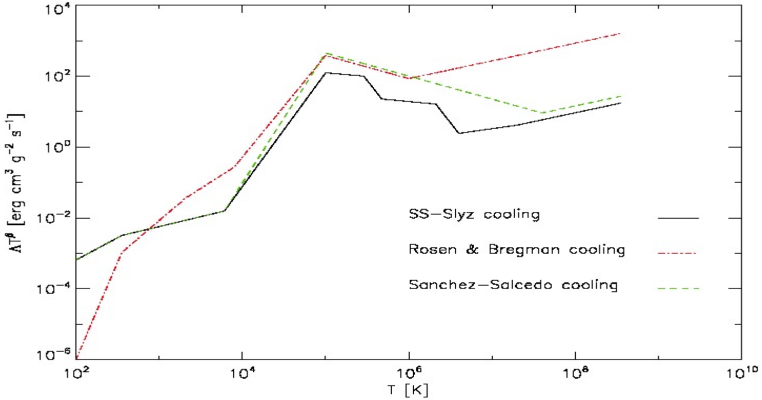


Figure: Cooling curves from (Rosen et al. 1993) plotted with dashed-dotted linestyle, (Sánchez-Salcedo et al. 2002) dashed, and (Slyz et al. 2005) solid.

Three phase ISM: thermal instability conditions ((Field 1965)) $\beta < 1$ (isobaric), $\beta < 0$ (isochoric) and $\beta < -3/2$ (isentropic) for an adiabatic index $\gamma = 5/3$

Filling factors of phases by height

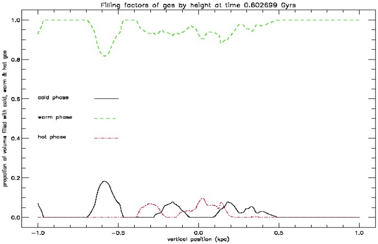


Figure: Filling factor of temperature phase by z. Prandtl:1 Cooling:SS

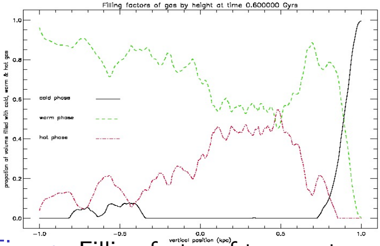


Figure: Filling factor of temperature phase by z. Prandtl:40 Cooling:SS

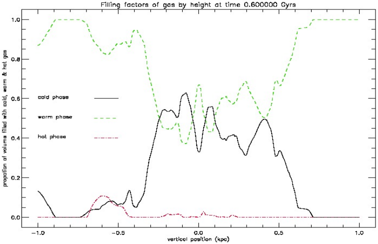


Figure: Filling factor of temperature phase by z. Prandtl:1 Cooling:RB

- cold phase: $T < 2000$ K
- warm phase $2000\text{K} \geq \mathrm{T} > 10^5$ K
- hot phase $T \geq 10^5$ K

Probability density functions

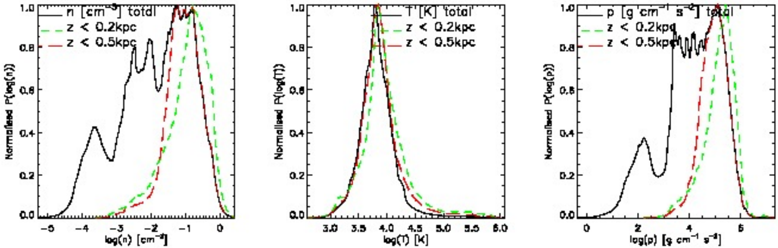
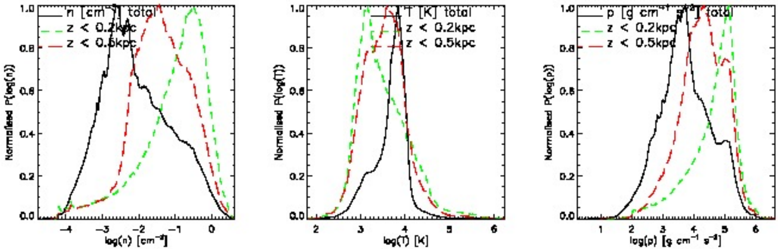


Figure: 615 Myr snapshot SS cooling (above), RB (below); density (left), temperature (middle), pressure (right); ISM Total, $z < 200$pc, $z < 500$pc



Conflicting outcomes

- Compare the dynamics for temperature and density with varying Prandtl numbers
- Handling mass loss - temperature and mass gradients - boundary losses
- Resolving cooling in shocks - snowplough/time step
- Handling data and resource management - storage, idl, cpu hours

Preliminary results: hydrodynamic model

- Comparsions with other models:
  (Gressel et al. 2008),
  (de Avillez & Breitschwerdt 2007),
  (Korpi et al. 1999),
  (Hanasz et al. 2006),
  (Ryan Joung & Mac Low 2006)
- Run parameters: Shear,
  Rotation,
  SN rate,
  Thermal conduction,
  Resolution
- Anisotropy,
  Phase characteristics,
  Correlation

Prospects

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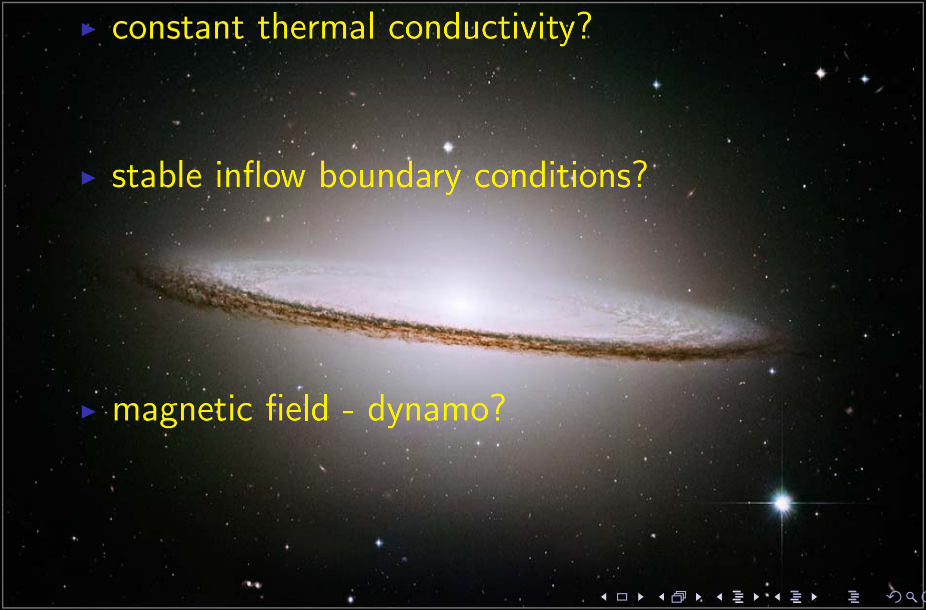

- constant thermal conductivity?
- stable inflow boundary conditions?
- magnetic field - dynamo?

Acknowledgements

Figure: The pencil-code
http://pencil-code.googlecode.com/

Figure: The work has been performed under the HPC-EUROPA2 project (project number: 228398) with the support of the European Commission - Capacities Area - Research Infrastructures

- UKMHD cluster: preliminary code development
  http://www-solar.mcs.st-andrews.ac.uk/cluster/
- (Korpi et al. 1999)
- (Gressel et al. 2008)

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