Flux tube in stratified solar atmosphere: magnetohydrostatic equilibrium

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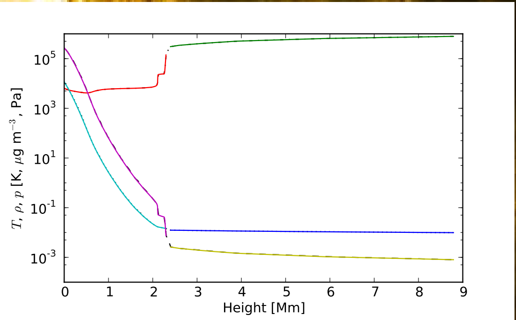
Atmospheric stratification

Title and background images courtesy SDO/NASA

Coronal heating problem?

(Vernazza et al. 1981)

(McWhirter et al. 1975)



Self-similar expanding flux tube and pressure balance

$$B_r = -\frac{\partial f}{\partial z} B_{0z} G, \quad B_\phi = 0, \quad B_z = \frac{\partial f}{\partial r} B_{0z} G, \qquad (1)$$

$\nabla \cdot \boldsymbol{B} = 0$ for $f = rB_{0z}$ and arbitrary $B_{0z} = B_{0z}(z)$ or $G = G(f)$.
(Schlüter & Temesváry 1958, Deinzer 1965, Gibson & Low 1998, Schüssler & Rempel 2005, Gordovskyy & Jain 2007)

$$f = r b_{00} \exp\left(-\frac{z}{z_1}\right), \quad G = \frac{2}{\sqrt{\pi} f_0} \exp\left[-\left(\frac{f}{f_0}\right)^2\right] \qquad (2)$$

$$\nabla p + \nabla \frac{|\boldsymbol{B}|^2}{2} + (\boldsymbol{B} \cdot \nabla)\boldsymbol{B} = \rho \boldsymbol{g}, \qquad (3)$$

(Fedun et al. 2011) Integrated numerically to find p and then ρ – turns out can be solved analytically. . .

Solving for corrections to p and ρ

$$\nabla(p_{\mathrm{h}} + p_{\mathrm{m}}) + \nabla\frac{|\boldsymbol{B}|^2}{2} + (\boldsymbol{B}\cdot\nabla)\boldsymbol{B} = (\rho_{\mathrm{h}} + \rho_{\mathrm{m}})\boldsymbol{g}, \tag{4}$$

Separate hydrostatic and then correct for magnetohydrostatic effect.

$$p_{\mathrm{h}} = P_{\mathrm{ref}}(0) + g\int_0^z \rho_{\mathrm{h}}(z^*)\mathrm{d}z^* \tag{5}$$

Interpolation(Vernazza et al. 1981, McWhirter et al. 1975)

$$\frac{\partial p_{\mathrm{m}}}{\partial r} = \quad -\frac{\partial}{\partial r}\frac{|\boldsymbol{B}|^2}{2} - B_r\frac{\partial B_r}{\partial r} - B_z\frac{\partial B_r}{\partial z} \tag{6}$$

$$\frac{\partial p_{\mathrm{m}}}{\partial z} = \rho_{\mathrm{m}} g \quad -\frac{\partial}{\partial z}\frac{|\boldsymbol{B}|^2}{2} - B_r\frac{\partial B_z}{\partial r} - B_z\frac{\partial B_z}{\partial z} \tag{7}$$

(8)

Solving for corrections to p and ρ

$$\nabla(p_{\rm h}+p_{\rm m})+\nabla\frac{|\boldsymbol{B}|^2}{2}+(\boldsymbol{B}\cdot\nabla)\boldsymbol{B}=(\rho_{\rm h}+\rho_{\rm m})\boldsymbol{g}, \tag{4}$$

Separate hydrostatic and then correct for magnetohydrostatic effect.

$$p_{\rm h}=P_{\rm ref}(0)+g\int_0^z\rho_{\rm h}(z^*)\mathrm{d}z^* \tag{5}$$

Interpolation(Vernazza et al. 1981, McWhirter et al. 1975)

$$\frac{\partial p_{\rm m}}{\partial r} = -\frac{\partial}{\partial r}\frac{|\boldsymbol{B}|^2}{2}-B_r\frac{\partial B_r}{\partial r}+\frac{\partial f}{\partial r}\frac{\partial^2 f}{\partial z^2}G^2+\frac{\partial f}{\partial r}\frac{\partial f}{\partial z}\frac{\partial G}{\partial z}G \tag{6}$$

$$\frac{\partial p_{\rm m}}{\partial z} = \rho_{\rm m}g-\frac{\partial}{\partial z}\frac{|\boldsymbol{B}|^2}{2}-B_r\frac{\partial B_z}{\partial r}-B_z\frac{\partial B_z}{\partial z} \tag{7}$$

(8)

Solving for corrections to p and ρ

$$\nabla(p_{\mathrm{h}} + p_{\mathrm{m}}) + \nabla\frac{|\boldsymbol{B}|^2}{2} + (\boldsymbol{B}\cdot\nabla)\boldsymbol{B} = (\rho_{\mathrm{h}} + \rho_{\mathrm{m}})\boldsymbol{g}, \tag{4}$$

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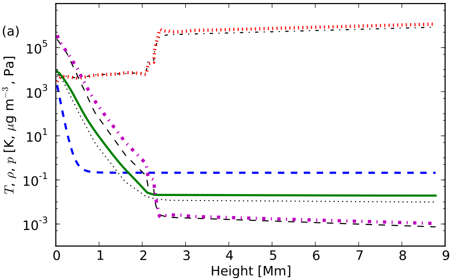
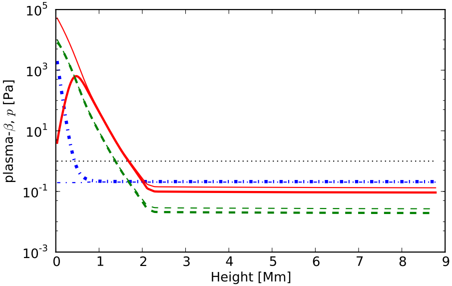
$$\rho_{\mathrm{m}} g - B_r \frac{\partial B_z}{\partial r} - B_z \frac{\partial B_z}{\partial z} + \frac{\partial}{\partial z}\left(\int B_r \frac{\partial B_r}{\partial r} + B_z \frac{\partial B_r}{\partial z} \mathrm{d}r\right) = 0 \tag{8}$$

Physical constraints



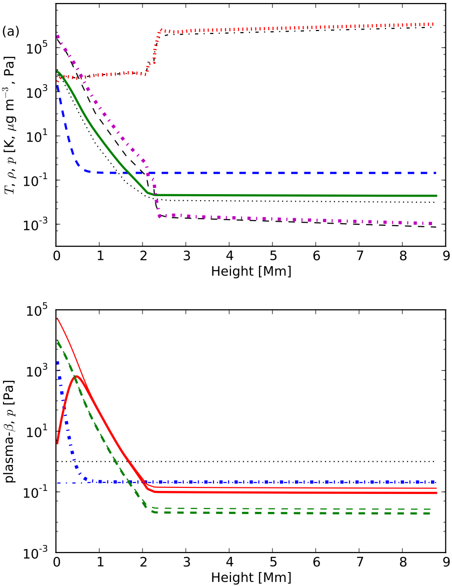


(Aschwanden 2005, Khomenko et al. 2008)

$$f = r\ b_{00} \exp\left(-\frac{z}{z_1}\right)$$

$$\text{plasma}-\beta = 2\frac{p}{|\boldsymbol{B}|^2}$$

Physical constraints

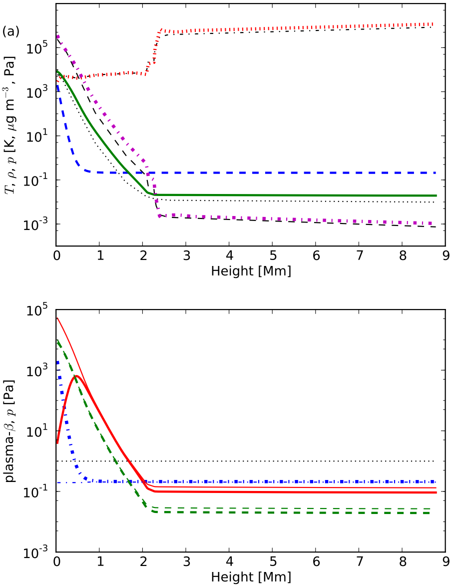


(Aschwanden 2005, Khomenko et al. 2008)

$$f = r\left[b_{01}\exp\left(-\frac{z}{z_1}\right) + b_{02}\exp\left(-\frac{z}{z_2}\right)\right]$$

$$\text{plasma}-\beta = 2\frac{p}{|\boldsymbol{B}|^2}$$

Physical constraints



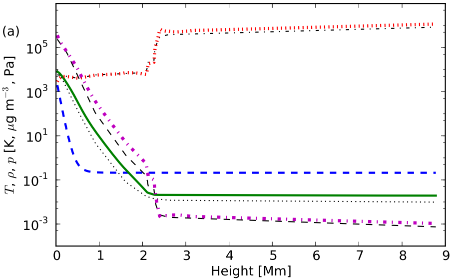
(Aschwanden 2005, Khomenko et al. 2008)

$$f = r\, B_{0z}$$

$$G = \frac{2}{\sqrt{\pi} f_0} \exp\left[- \left(\frac{f}{f_0} \right)^2 \right]$$

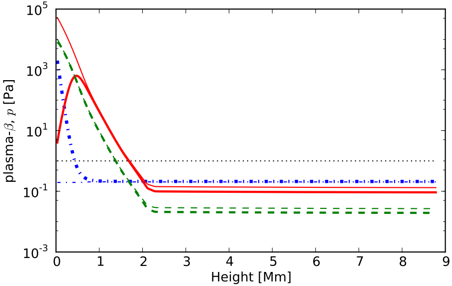
$$\text{plasma}-\beta = 2 \frac{p}{|\boldsymbol{B}|^2}$$

Physical constraints



$$f = r\,B_{0z}$$

$$G' = \frac{2B_{0z}}{\sqrt{\pi}f_0} \exp\left[-\left(\frac{rB_{0z}}{f_0}\right)^2\right]$$



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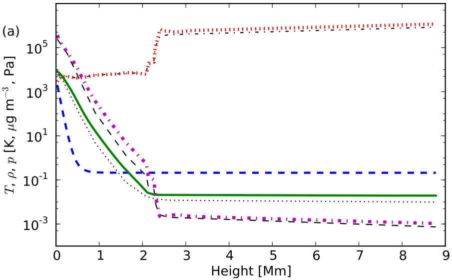
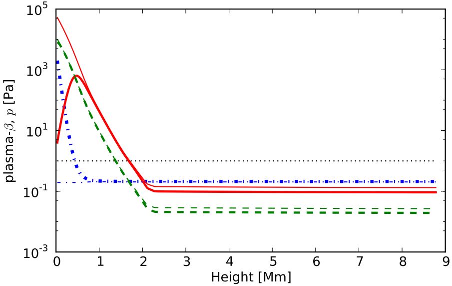
(Aschwanden 2005, Khomenko et al. 2008)

Physical constraints





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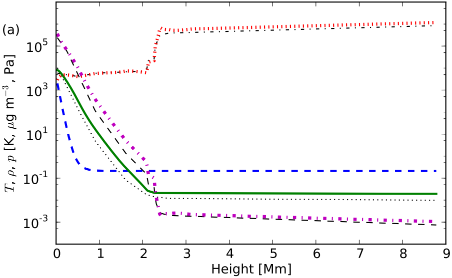

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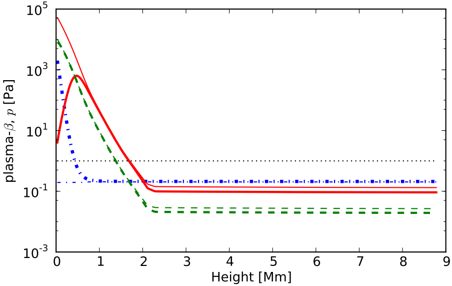
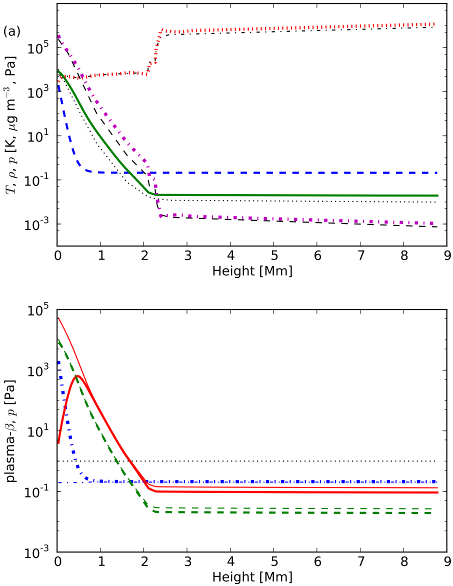


$$\text{plasma}-\beta = 2\frac{p}{|\boldsymbol{B}|^2}$$

(Aschwanden 2005, Khomenko et al. 2008)

Physical constraints



(Aschwanden 2005, Khomenko et al. 2008)

$$f = r\, B_{0z}$$

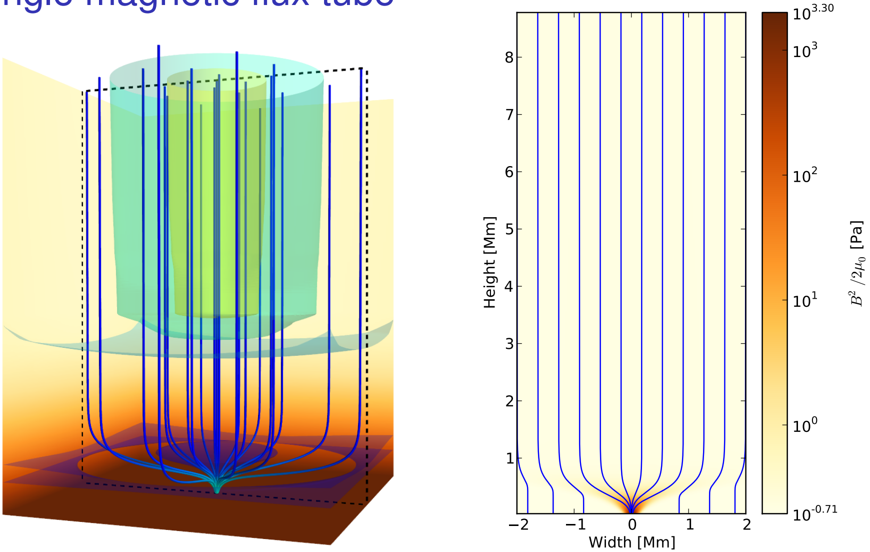
$$G' = \frac{2B_{0z}}{\sqrt{\pi} f_0} \exp\left[-\left(\frac{f}{f_0} \right)^2 \right]$$

$$\text{plasma}-\beta = 2 \frac{p}{|\boldsymbol{B}|^2}$$

$$B_r = -\frac{\partial f}{\partial z} B_{0z} G + \frac{r B_{bz}}{z_b}$$

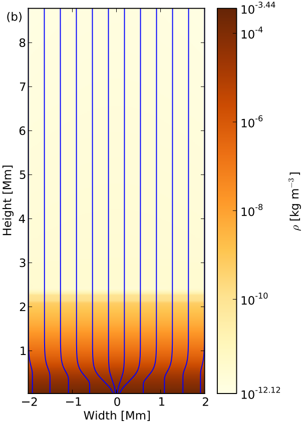
$$B_z = \frac{\partial f}{\partial r} B_{0z} G + 2B_{bz}$$

Single magnetic flux tube



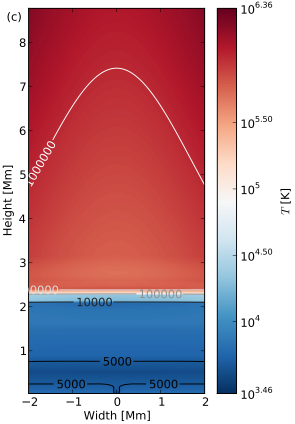
B-field lines (blue), plasma$-\beta$ (purple – green isosurfaces), p_{thermal} (3D rear and bottom surfaces), p_{magnetic} (2D fill)

Single magnetic flux tube



ρ

(***B***-field lines – blue)



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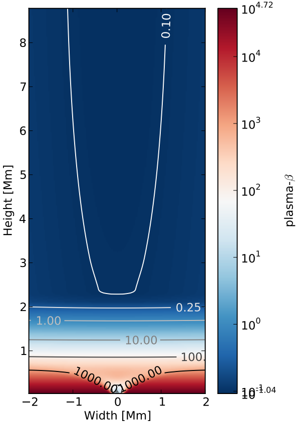


plasma$-\beta$

Single magnetic flux tube - pressure terms

submitted MNRAS 17th May, 2013 `http://arxiv.org/abs/1305.4788`

$$
\begin{aligned}
p_{\mathrm{m}} \quad = \quad & -\frac{|\boldsymbol{B}|^2}{2} + \left(\frac{2B_{bz} f^2 G}{{B_{0z}}^2} + \frac{B_{bz} {f_0}^2 G}{{B_{0z}}^2} \right) \frac{\partial B_{0z}}{\partial z}^2 \\
& - \frac{B_{bz} {f_0}^2 G}{B_{0z}} \frac{\partial^2 B_{0z}}{\partial z^2} - \frac{B_{0z} {f_0}^2 G^2}{4} \frac{\partial^2 B_{0z}}{\partial z^2} - \frac{r^2}{2} \frac{\partial B_{bz}}{\partial z}^2 \\
& - \frac{f^2 G}{B_{0z}} \frac{\partial B_{bz}}{\partial z} \frac{\partial B_{0z}}{\partial z} - \frac{{f_0}^2 G}{2} \frac{\partial^2 B_{bz}}{\partial z^2} + r^2 B_{bz} \frac{\partial^2 B_{bz}}{\partial z^2}.
\end{aligned}
$$

Single magnetic flux tube - pressure terms

submitted MNRAS 17th May, 2013 http://arxiv.org/abs/1305.4788

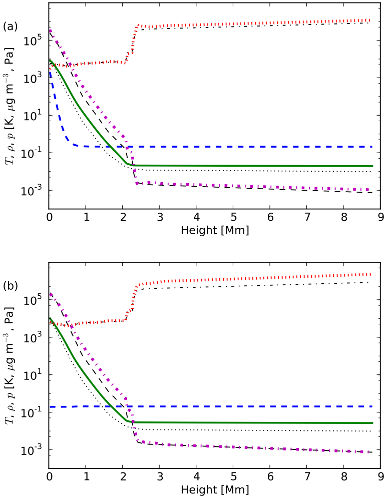
$$p_{\mathrm{m}} = -\frac{|\boldsymbol{B}|^2}{2}$$

$$-\frac{B_{0z} {f_0}^2 G^2}{4} \frac{\partial^2 B_{0z}}{\partial z^2}$$

Single magnetic flux tube - density terms

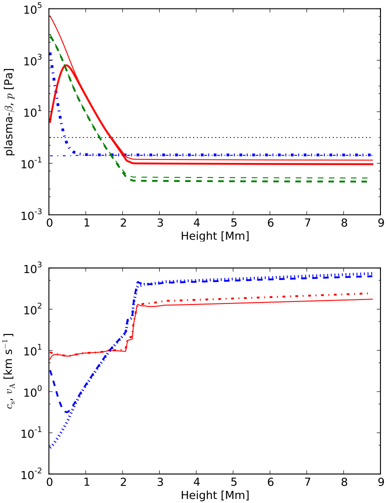
$$
\begin{aligned}
\int \mathrm{d}z\rho_{\mathrm{m}} g \quad & - \left[\frac{2{B_{0z}}^4 G r^2}{{f_0}^2} + 2{B_{0z}}^2 G - 4B_{bz}\right]\frac{\partial B_{bz}}{\partial z} \\
& + \frac{{f_0}^2 G}{2}\frac{\partial^3 B_{bz}}{\partial z^3} + B_{bz} r^2 \frac{\partial^3 B_{bz}}{\partial z^3} - 2{B_{0z}}^3 G^2 \frac{\partial B_{0z}}{\partial z} \\
& - 4B_{bz}B_{0z}G\left[1 - \frac{f^2}{{f_0}^2}\right]\frac{\partial B_{0z}}{\partial z} + \frac{{f_0}^2 G}{B_{0z}}\frac{\partial B_{bz}}{\partial z}\frac{\partial^2 B_{0z}}{\partial z^2} \\
& + fGr\frac{\partial B_{bz}}{\partial z}\frac{\partial^2 B_{0z}}{\partial z^2} - \frac{3B_{bz}{f_0}^2 G}{{B_{0z}}^2}\frac{\partial B_{0z}}{\partial z}\frac{\partial^2 B_{0z}}{\partial z^2} \\
& + \left[\frac{{f_0}^2 G^2}{4} - f^2 G^2 - 6B_{bz}Gr^2\right]\frac{\partial B_{0z}}{\partial z}\frac{\partial^2 B_{0z}}{\partial z^2} \\
& - \left[\frac{2f^2 G r^2}{{f_0}^2} + \frac{{f_0}^2 G}{{B_{0z}}^2} + Gr^2\right]\frac{\partial B_{bz}}{\partial z}\frac{\partial B_{0z}}{\partial z}^2 \\
& + B_{bz}G\left[\frac{r^2}{B_{0z}} + \frac{2{f_0}^2}{{B_{0z}}^3} + \frac{4fr^3}{{f_0}^2}\right]\frac{\partial B_{0z}}{\partial z}^3 \\
& + \frac{B_{0z}{f_0}^2 G^2}{4}\frac{\partial^3 B_{0z}}{\partial z^3} + \frac{B_{bz}{f_0}^2 G}{B_{0z}}\frac{\partial^3 B_{0z}}{\partial z^3} = 0.
\end{aligned}
$$

Flux tube structure



Upper: along flux tube axis, lower: outside flux tube

[T-red, ρ-purple, p_t-green, p_m-blue]

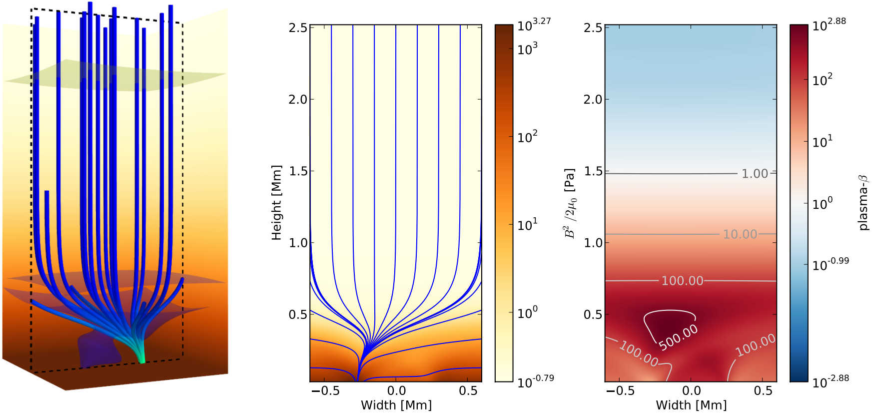


Upper: plasma-β-red, p_t-green, p_m-blue,

Lower: c_S-red, v_A-blue

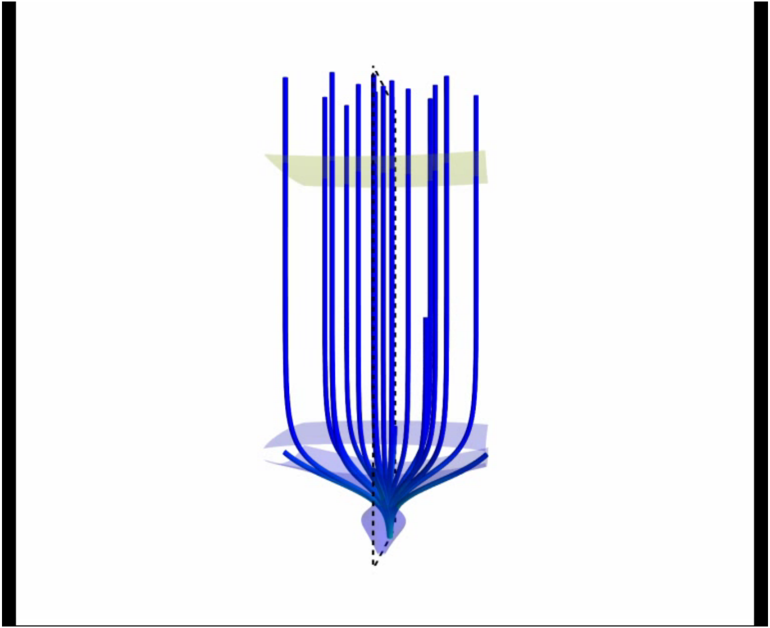
[axis (think) outside (thin)]

Multiple magnetic flux tube



$\boldsymbol{B}$-field lines (blue), plasma$-\beta$ (purple – green isosurfaces), p_{thermal} (3D rear and bottom surfaces), p_{magnetic} (2D fill), plasma-β (red – blue)

Multiple magnetic flux tube



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